# Equations with Two Unknowns

**Prior Knowledge:**

- Substitution into an equation.

### Example 1:

Find three pairs of values of $a$ and $b$ for the following equation: $2a + b = 15$

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In this question, we have an equation with two unknown numbers, $a$ and $b$. We have to find **pairs** of values for $a$ and $b$ which make the equation true. We don't have enough information to be sure of their exact values but we do have enough information to list possibilities.

To start, we can't just pick a random pair of numbers. If we say $a = 3$ and $b = 12$, we get:

$2 \times 3 + 12 = 15$

$6 + 12 = 15$

$18 = 15$ ✗

This is obviously wrong. Once we pick a value for $a$, there is only one possible value of $b$ to pair with it. Let's say $a = 5$:

$2 \times 5 + b = 15$

$10 + b = 15$

In this case, if $a$ is 5 then $b$ must also be 5. Our pair of values are:

**$a = 5$, $b = 5$**

This also works the other way round. Let's start by saying $b = 1$:

$2a + 1 = 15$

Now, we know something plus 1 is 15. That something must be 14:

$2a + 1 = 15$

$14 + 1 = 15$

$2a = 14$

If 2 lots of $a$ give 14, 1 lot of $a$ must be 7. Again, we have a pair of values:

**$a = 7$, $b = 1$**

We can then substitute this back into our original equation, to check it works:

$2 \times 7 + 1 = 15$

$14 + 1 = 15$

$15 = 15$ ✓

Our question asked for three pairs of values, so we need one more. This time, we'll use a negative number. Let's say $a$ = -1:

$2 \times -1 + b = 15$

$-2 + b = 15$

We have to add 17 to -2 to get 15, so $b$ = 17. We have our third pair of values:

**$a$ = -1, $b$ = 17**

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**Example 2:**

John goes to a cafe. He buys two cups of coffee and three cups of tea. He spends £3.50. Suggest a pair of prices for tea and coffee.

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This question looks very different but it is asking the same thing. We need to find a valid pair of prices for tea and coffee so that two cups of coffee and three cups of tea cost £3.50.

We can work this through verbally or algebraically. Let's start by thinking of a price for a cup of coffee: £1.

This means two cups of coffee cost £2.

If we take £2 from £3.50, we get £1.50, which is the amount John must have spent on three cups of tea.

If we divide £1.50 by three, we will get the price of one cup of tea: £0.50 or 50p.

This means our pair of prices are: a cup of coffee costs £1 and a cup of tea costs 50p.

Now, let's look at it algebraically. If we say that the price of a cup of coffee is $C$ and the price of a cup of tea is $T$, we get:

$2C + 3T = 3.50$

This time, let's say a cup of coffee costs £0.85:

$2 \times 0.85 + 3T = 3.50$

$1.70 + 3T = 3.50$

If we subtract 1.70 from 3.50, we get the value of $3T$:

$3T = 1.80$

We know $3T$ is 1.80 so, to get $T$, we divide by 3:

$T$ = £0.60

We have another pair of values: a cup of coffee costs £0.85 (or 85p) and a cup of tea costs £0.60 (or 60p).

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**$2 \times 3 + 12 = 15$**

**$6 + 12 = 15$**

**$18 = 15$ ✗**

**This is obviously wrong. Once we pick a value for $a$, there is only one possible value of $b$ to pair with it. Let's say $a = 5$:**

**$2 \times 5 + b = 15$**

**$10 + b = 15$**

**In this case, if $a$ is 5 then $b$ must also be 5. Our pair of values are:**

**$a = 5$, $b = 5$**

**This also works the other way round. Let's start by saying $b$ = 1:**

**$2a$ + 1 = 15**

**Now, we know *something* plus 1 is 15. That something must be 14:**

**$2a$ + 1 = 15**

**14 + 1 = 15**

**$2a$ = 14**

**If 2 lots of $a$ give 14, one lot of $a$ must be 7. Again, we have a pair of values:**

**$a$ = 7, $b$ = 1**

**We can then substitute this back into our original equation, to check it works:**

**2 × 7 + 1 = 15**

**14 + 1 = 15**

**15 = 15 ✓**

**Our question asked for three pairs of values, so we need one more. This time, we'll use a negative number. Let's say $a$ = -1:**

**2 × -1 + $b$ = 15**

**-2 + $b$ = 15**

**We have to add 17 to -2 to get 15, so $b$ = 17. We have our third pair of values:**

**$a$ = -1, $b$ = 17**

## Example 2:

**John goes to a cafe. He buys two cups of coffee and three cups of tea. He spends £3.50. Suggest a pair of prices for tea and coffee.**

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**This question looks very different but it is asking the same thing. We need to find a valid pair of prices for tea and coffee so that two cups of coffee and three cups of tea cost £3.50.**

**We can work this through verbally or algebraically. Let's start by thinking of a price for a cup of coffee: £1.**

**This means two cups of coffee cost £2.**

**If we take £2 from £3.50, we get £1.50, which is the amount John must have spent on three cups of tea.**

**If we divide £1.50 by three, we will get the price of one cup of tea: £0.50 or 50p.**

**This means our pair of prices are: a cup of coffee costs £1 and a cup of tea costs 50p.**

**Now, let's look at it algebraically. If we say that the price of a cup of coffee is $C$ and the price of a cup of tea is $T$, we get:**

$2C + 3T = 3.50$

**This time, let's say a cup of coffee costs £0.85:**

$2 \times 0.85 + 3T = 3.50$

$1.70 + 3T = 3.50$

**If we subtract 1.70 from 3.50, we get the value of $3T$:**

$3T = 1.80$

**We know $3T$ is 1.80 so, to get $T$, we divide by 3:**

$T$ = £0.60

**We have another pair of values: a cup of coffee costs £0.85 (or 85p) and a cup of tea costs £0.60 (or 60p).**

# Your Turn

## Your Turn

1. **Jamie is finding possible values for equations with two unknowns. For each question, they have found the first unknown but not the second. Find each second unknown:**

| | | |
|---|---|---|
| **a.** | $a + b = 15$ | $a = 5, b =$ ____ |
| **b.** | $x \times y = 15$ | $x = 1, y =$ ____ |
| **c.** | $2a + b = 10$ | $a = 4, b =$ ____ |
| **d.** | $3a + 2b = 12$ | $a = 2, b =$ ____ |
| **e.** | $2a + 2b = 6$ | $a = -1, b =$ ____ |

## Your Turn

2. **Choose a valid pair of values for each equation (remember to substitute your answers back into the equation to check that it works):**

   **a.** $a + b = 9$

   **b.** $c - d = 5$

   **c.** $e \times f = 20$

   **d.** $g \div h = 2$

   **e.** $2a + b = 10$

   **f.** $5a - b = 9$

   **g.** $2a + 4b = 20$

   **h.** $5a - 2b = 14$

   **i.** $8a - 3b = 21$

   **j.** $5a + 2b = -1$

3. **Choose a pair of values for the equation $2a + 5b = 3$ where $a$ is negative and $b$ is positive.**

4. **Choose a pair of values for $2a + 3b = 24$ where $a$ is a multiple of 3 and $b$ is a multiple of 4.**

5. **Choose a pair of values for $5a - 3b = 11$ where $a$ is a square number and $b$ is a prime number.**

## Your Turn

6. John buys 3 pencils and a rubber from the school shop. He spends 70p. List two possible pairs of prices for a pencil and a rubber.

7. Imran buys 3 cans of pop and 2 slices of pizza from the cafeteria. He spends £2. What is a possible pair of prices for cans of pop and slices
of pizza?

8. Sarah buys two cups of tea and three cups of coffee for exactly £1.50. She says each cup of tea costs 50p. Why must she be wrong?

9. Sandra is in charge of refurbishing a floor of a hotel. She has spent £6000 on furniture, which has been spent on beds and cabinets for each room. If each room has 2 beds and 3 cabinets, and there are 10 rooms on the floor, give a pair of possible costs for beds and cabinets.

## Your Turn Challenge

**1 a.** $2x + 4y = 22$. Give three pairs of possible values for $x$ and $y$.

**b.** $3x - y = 5$. Give three pairs of possible values for $x$ and $y$.

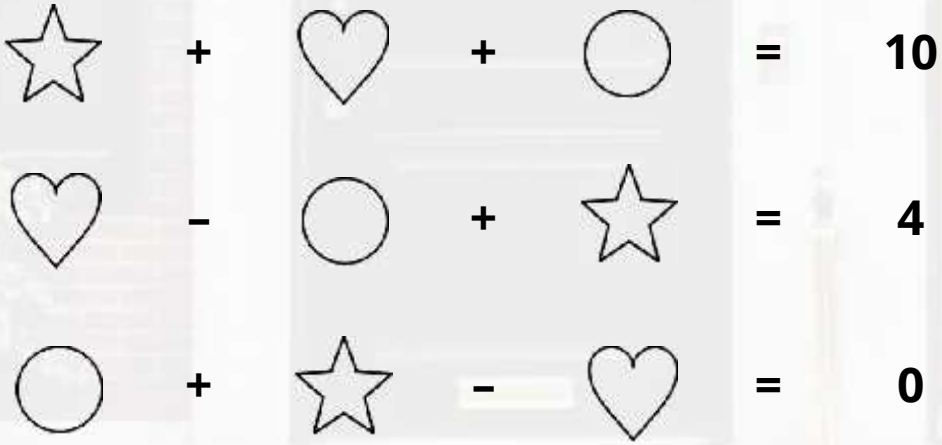
**c.** The equations in question a and b use the same values of $x$ and $y$. What must they be?

**d.** These two equations use the same values for a and b. What are they?

$2a + b = 17$

$a - 2b = -9$

**2.** Find the value of each symbol:



# Answers

## Answers

1. **Jamie is finding possible values for equations with two unknowns. For each question, they have found the first unknown but not the second. Find each second unknown:**

| | | |
|---|---|---|
| **a.** | $a + b = 15$ | $a = 5, b =$ **10** |
| **b.** | $x \times y = 15$ | $x = 1, y =$ **15** |
| **c.** | $2a + b = 10$ | $a = 4, b =$ **2** |
| **d.** | $3a + 2b = 12$ | $a = 2, b =$ **3** |
| **e.** | $2a + 2b = 6$ | $a = -1, b =$ **4** |

## Answers

**2. Choose a valid pair of values for each equation (remember to substitute your answers back into the equation to check that it works):**

| | | |
|---|---|---|
| **a.** | $a + b = 9$ | any pair of numbers adding to 9 |
| **b.** | $c - d = 5$ | any pair of numbers where $c$ is 5 more than $d$ |
| **c.** | $e \times f = 20$ | 1 and 20, 2 and 10, 4 and 5 or equivalent decimals/fractions |
| **d.** | $g \div h = 2$ | any pair of numbers where $g$ is double $h$ |
| **e.** | $2a + b = 10$ | e.g. $a = 1$ and $b = 8$, $a = 2$ and $b = 6$, etc |
| **f.** | $5a - b = 9$ | e.g. $a = 2$ and $b = 1$, $a = 3$ and $b = 6$, etc |
| **g.** | $2a + 4b = 20$ | e.g. $a = 2$ and $b = 4$, $a = 4$ and $b = 3$, etc |
| **h.** | $5a - 2b = 14$ | e.g. $a = 4$ and $b = 3$, $a = 6$ and $b = 8$, etc |
| **i.** | $8a - 3b = 21$ | e.g. $a = 3$ and $b = 1$, $a = 6$ and $b = 9$, etc |
| **j.** | $5a + 2b = -1$ | e.g. $a = 1$ and $b = -3$, $a = 3$ and $b = -8$, etc |

## Answers

3. **Choose a pair of values for the equation $2a + 5b = 3$ where $a$ is negative and $b$ is positive.**

   **e.g. $a = -1$, $b = 1$**

   **$a = -6$, $b = 3$**

   **$a = -11$, $b = 5$**

4. **Choose a pair of values for $2a + 3b = 24$ where $a$ is a multiple of 3 and $b$ is a multiple of 4.**

   **e.g. $a = 6$, $b = 4$**

   **$a = 12$, $b = 0$**

   **$a = 18$, $b = -4$**

5. **Choose a pair of values for $5a - 3b = 11$ where $a$ is a square number and $b$ is a prime number.**

   **e.g. $a = 4$, $b = 3$**

   **$a = 16$, $b = 23$**

   **$a = 64$, $b = 103$**

## Answers

6. John buys 3 pencils and a rubber from the school shop. He spends 70p. List two possible pairs of prices for a pencil and a rubber.

   e.g. 1p and 67p, 5p and 55p, 10p and 40p, 20p and 10p, etc

7. Imran buys 3 cans of pop and 2 slices of pizza from the cafeteria. He spends £2. What is a possible pair of prices for cans of pop and slices

   of pizza?

   e.g. 50p and 25p, £0.40 and £0.40, 20p and £0.70, etc

8. Sarah buys two cups of tea and three cups of coffee for exactly £1.50. She says each cup of tea costs 50p. Why must she be wrong?

   Two cups of tea add up to £1, so 50p is left for three cups of coffee.

   50 ÷ 3 = 16.666...p, which is not a possible price.

## Answers

9. **Sandra is in charge of refurbishing a floor of a hotel. She has spent £6000 on furniture, which has been spent on beds and cabinets for each room. If each room has 2 beds and 3 cabinets, and there are 10 rooms on the floor, give a pair of possible costs for beds and cabinets.**

   **Sandra has to buy 20 beds and 30 cabinets, and has spent £6000. Possible pairs of values include £150 and £100, £75 and £150, £225 and £50, etc.**

## Challenge Answers

**1 a.** **$2x + 4y = 22$. Give three pairs of possible values for $x$ and $y$.**

**e.g. $x = 1$, $y = 5$**

**$x = 3$, $y = 4$**

**$x = 5$, $y = 3$, etc**

**b.** **$3x - y = 5$. Give three pairs of possible values for $x$ and $y$.**

**e.g. $x = 2$, $y = 1$**

**$x = 3$, $y = 4$**

**$x = 4$, $y = 7$, etc**

**c.** **The equations in question a and b use the same values of $x$ and $y$. What must they be?**

**You must find a pair of values that works for the equation in question a and the equation in question b. There is only one: $x = 3$, $y = 4$.**

**d.** **These two equations use the same values for a and b. What are they?**

**$2a + b = 17$**

**$a - 2b = -9$**

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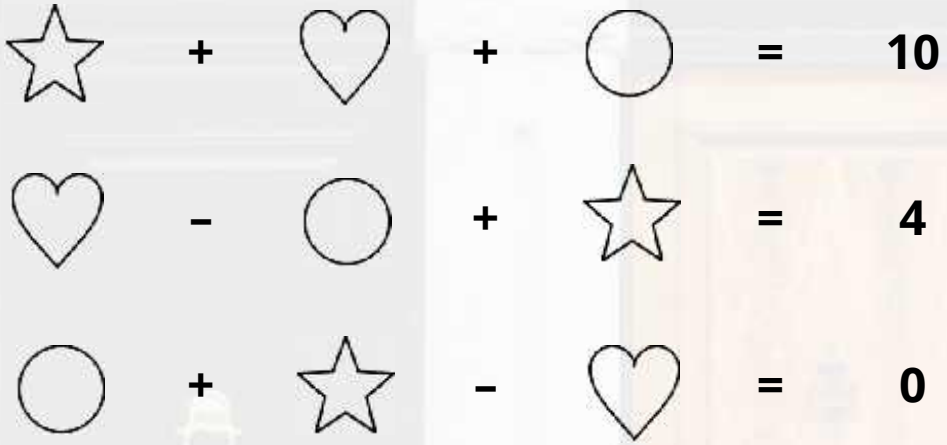
## Challenge Answers

**2. Find the value of each symbol:**




 = 2

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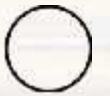
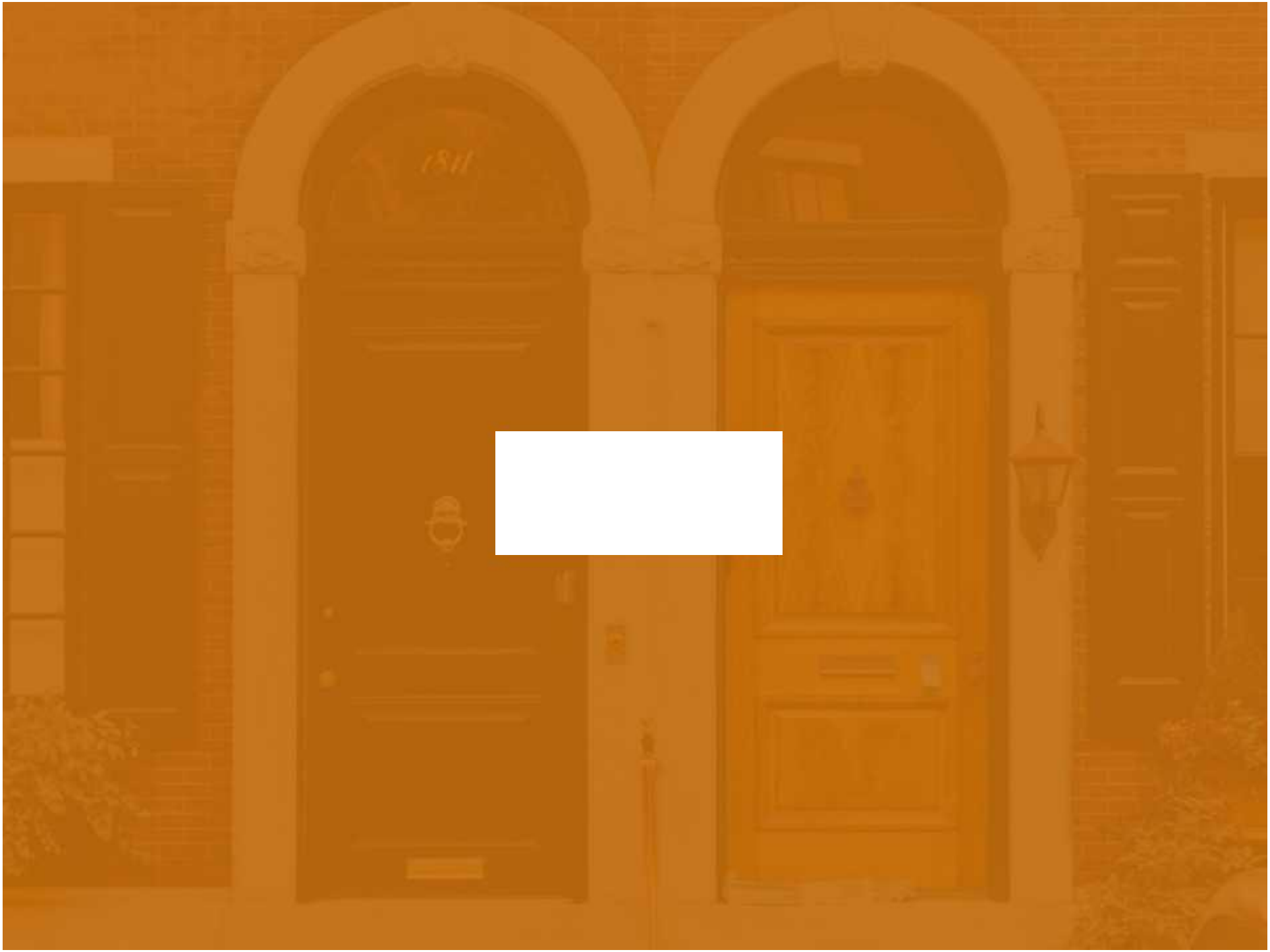
 = 3



# Equations with Two Unknowns **Answers**

1. Jamie is finding possible values for equations with two unknowns. For each question, they have found the first unknown but not the second. Find each second unknown:

| | | | |
|---|---|---|---|
| a. | $a + b = 15$ | $a = 5, b =$ ____ | **$b = 10$** |
| b. | $x \times y = 15$ | $x = 1, y =$ ____ | **$y = 15$** |
| c. | $2a + b = 10$ | $a = 4, b =$ ____ | **$b = 2$** |
| d. | $3a + 2b = 12$ | $a = 2, b =$ ____ | **$b = 3$** |
| e. | $2a + 2b = 6$ | $a = -1, b =$ ____ | **$b = 4$** |

2. Choose a valid pair of values for each equation (remember to substitute your answers back into the equation to check that it works):

| | | |
|---|---|---|
| a. | $a + b = 9$ | **any pair of numbers adding to 9** |
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| i. | $8a - 3b = 21$ | **e.g. $a = 3$ and $b = 1$, $a = 6$ and $b = 9$, etc** |
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3. Choose a pair of values for the equation $2a + 5b = 3$ where $a$ is negative and $b$ is positive.

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   **$a = -11$, $b = 5$**

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**Challenge:**

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   **$x = 4$, $y = 7$, etc**

c. The equations in question **a** and **b** use the same values of $x$ and $y$. What must they be?

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$2a + b = 17$

$a - 2b = -9$

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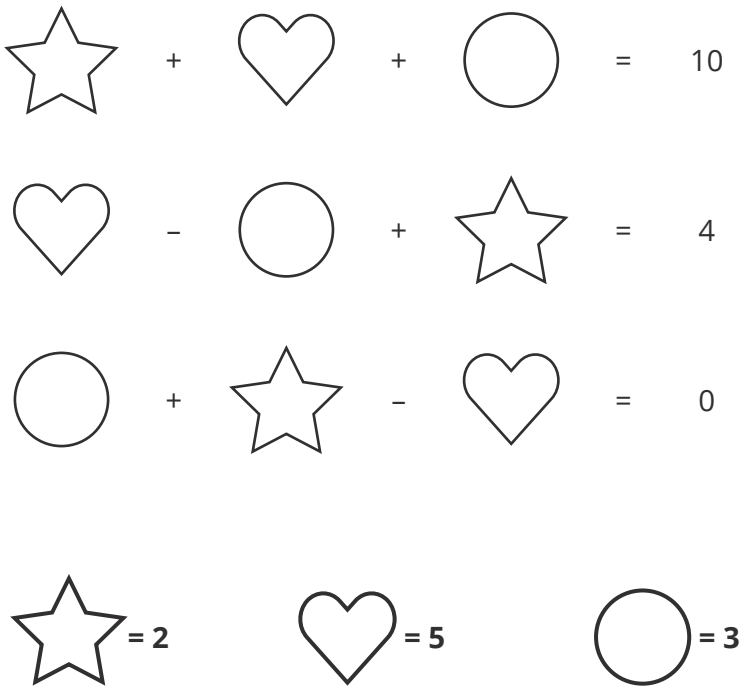
2. Find the value of each symbol:



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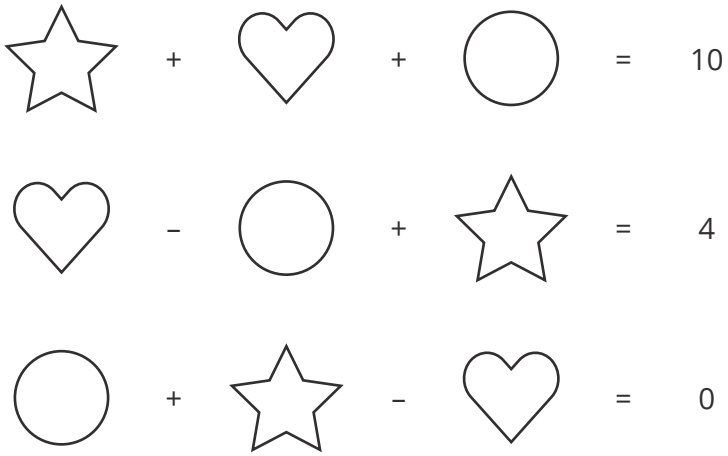

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   c. $2a + b = 10$ $a = 4$, $b =$ _____ _____

   d. $3a + 2b = 12$ $a = 2$, $b =$ _____ _____

   e. $2a + 2b = 6$ $a = -1$, $b =$ _____ _____

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   _____

   b. $c - d = 5$

   _____

   c. $e \times f = 20$

   _____

   d. $g \div h = 2$

   _____

   e. $2a + b = 10$

   _____

   f. $5a - b = 9$

   _____

   g. $2a + 4b = 20$

   _____

   h. $5a - 2b = 14$

   _____

   i. $8a - 3b = 21$

   _____

   j. $5a + 2b = -1$

   _____

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7. Imran buys 3 cans of pop and 2 slices of pizza from the cafeteria. He spends £2. What is a possible pair of prices for cans of pop and slices of pizza?

8. Sarah buys two cups of tea and three cups of coffee for exactly £1.50. She says each cup of tea costs 50p. Why must she be wrong?

9. Sandra is in charge of refurbishing a floor of a hotel. She has spent £6000 on furniture, which has been spent on beds and cabinets for each room. If each room has 2 beds and 3 cabinets, and there are 10 rooms on the floor, give a pair of possible costs for beds and cabinets.

**Challenge:**

1. a. $2x + 4y = 22$. Give three pairs of possible values for $x$ and $y$.

b. $3x - y = 5$. Give three pairs of possible values for $x$ and $y$.

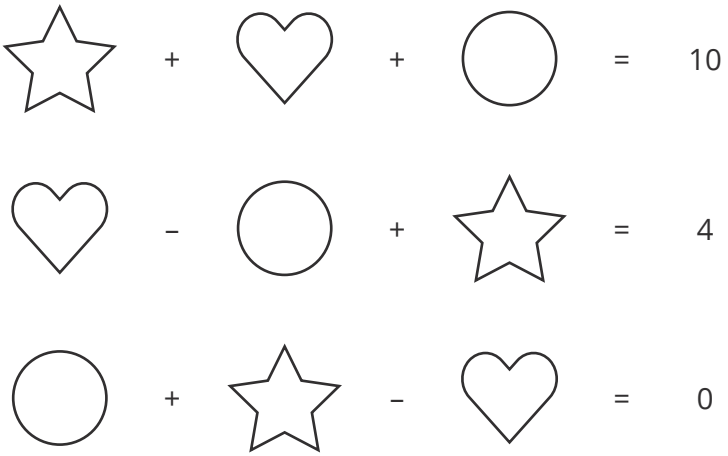
c. The equations in question **a** and **b** use the same values of $x$ and $y$. What must they be?

d. These two equations use the same values for $a$ and $b$. What are they?

$2a + b = 17$

$a - 2b = -9$

2. Find the value of each symbol:

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